

## Day 5 - PM

Ex: "Is the vector  $\begin{bmatrix} -3 \\ 8 \\ 1 \end{bmatrix}$  in  $\text{Span} \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ -13 \\ -3 \end{bmatrix} \right\}$ ?"

" $\text{Span} \{a_1, a_2\}$  is a plane through the origin in  $\mathbb{R}^3$ , is  $b$  in that plane?"

"Does the equation  $x_1 a_1 + x_2 a_2 = b$  have a solution?"

→ Row reduce the augmented matrix!

$$\left[ \begin{array}{cc|c} 1 & 5 & -3 \\ 2 & -13 & 8 \\ 3 & -3 & 1 \end{array} \right] \xrightarrow{\substack{2R_1 + R_2 \\ 3R_1 + R_3}} \left[ \begin{array}{cc|c} 1 & 5 & -3 \\ 0 & -3 & 2 \\ 0 & -18 & 10 \end{array} \right] \xrightarrow{-1/3 R_2} \left[ \begin{array}{cc|c} 1 & 5 & -3 \\ 0 & 1 & -2/3 \\ 0 & -18 & 10 \end{array} \right]$$

$$\xrightarrow{\substack{-5R_2 + R_1 \\ 18R_2 + R_3}} \left[ \begin{array}{cc|c} 1 & 0 & 1/3 \\ 0 & 1 & -2/3 \\ 0 & 0 & -2 \end{array} \right] \Rightarrow 0 = -2 \text{ No Solution}$$

$b$  is not in  $\text{Span} \{a_1, a_2\}$

$\text{Col } A = \left\{ \vec{b} : \vec{b} = A\vec{x} \text{ for some } \vec{x} \in \mathbb{R}^n \right\}$

Ex: find a matrix  $A$  such that  $W = \text{Col } A$

$$W = \left\{ \begin{bmatrix} 6a-b \\ a+b \\ -7a \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

Write  $W$  as a linear combination

$$W = \left\{ a \begin{bmatrix} 6 \\ 1 \\ -7 \end{bmatrix} + b \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} : a, b \in \mathbb{R} \right\} = \text{Span} \left\{ \begin{bmatrix} 6 \\ 1 \\ -7 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Use the vectors in the spanning set as columns of  $A$

$$A = \begin{bmatrix} 6 & -1 \\ 1 & 1 \\ -7 & 0 \end{bmatrix}$$





Ex: Let  $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$  and  $u = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$

is  $u \in \text{Nul} A$ ?

( $u \in \text{Nul} A$  if  $A\vec{u} = \vec{0}$ )

$$\begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

yes,  $u \in \text{Nul} A$